STATISTICAL APPROACH OF STRUCTURE-ACTIVITY RELATIONSHIPS: A CASE STUDY

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Structure/Activity/Property Relationships (SARs, SPRs, and PARs) appears with the studies of Louis Plack HAMMETT in 1937 [1]. The most important applications of Hammett's equation were summarized in [2]. Quantitative relationships (QSAR, QSPR, QPAR) occur when the property/activity is quantitative. Not all properties and activities of chemical compounds can be classified as quantitative. In fact, few properties meet all theoretical requirements to be quantitative [3]. From this reason in the last time are avoided to be used QSAR, QSPR, and QPAR, in their place being used (Q)SAR, (Q)SPR, and (Q)PAR, or more simple SAR, SPR, and PAR. Structure-based approaches have two levels (topological and geometrical). In the topological based level, an atom, a bond from a molecule can exist (and then are evidenced through electronic transitions and/or molecular vibrations) or not (being a matter of 0 and 1). Not so simple stays things related to molecular geometry (especially on liquid or gas phases). Heisenberg uncertainly principle [4] shows the uncertainly rules presented at micro level (molecular and atomic level). More than that, molecular geometry depends on the environment where the molecule is (vicinity of the molecule), temperature, pressure, so on, thus dealing with molecular geometry is both a matter of relativity and a matter of uncertainty. Thus, Structure-Property-Activity Relationships (SPARs) must deal with certainties (such as molecular topology), uncertainties (such as molecular geometry), relativities (such as biological activities) and evidences (such as physical and chemical quantitative properties). The Molecular Descriptors Family (MDF) is an original structure-based approach [5] which generates for given structure(s) a huge pool of quantum based [6] descriptors of structure (indices) using a unitary methodology [7] that incorporated both topological and geometrical approaches. SPARs MDF methodology [8] uses a genetic algorithm [9] in order to obtain so called MDF-SPARs (structure-property or structure-activity relationships with Molecular Descriptors Family members relating the structure). AIM: to assess the potential of MDF-SPARs for drug design. IDEA: to develop, test, and use a complete statistical methodology in the evaluation of obtained relationships, to estimate and predict the desired

Parameter	Mathematical formula	Remarks		General issues				
Simple	$r_{SP}=r(Y,MDF_i), p_{SP}=p(r_{SP},m,df=1)$	r _{SP} : correlation between Y and MDF _i ;		MLR: $\hat{Y} = \Sigma_i a_i MDF_i$;				
correlation		p _{SP} : probability of no linear dependence betw		a _i : real coefficients (MLR coefficie				
analysis		a larger p_{SP} (usually > 5%) leads to excluding	g of MDF _i from MLR equation	Ŷ: estimator of the measured activi				
Inter-correlation	$r_{IP}=r(MDF_i,MDF_j), p_{IP}=p(r_{IP},m,df=1)$	r_{IP} : correlation between MDF _i and MDF _j ;		MDF _i : an MDF member (an array	with m values);			
analysis		p _{IP} : probability of no linear dependence betw		m: sample size;				
		a larger r_{IP} (usually larger than r_{MP}) leads to a		n: number of variables;				
		a solution can be excluding of MDF _i (if r_{SP} (
		(if $r_{SP}(Y,MDF_i) < r_{SP}(Y,MDF_j)$ is false) from same procedure can be applied for $p_{MP} > p_{IP}$	MLR equation;	r: Pearson correlation coefficient;	ng either Fisher or Student distribution);			
Multiple	$r_{MP}=r(Y,\hat{Y}), p_{MP}=p(r_{MP},m,df=n)$	same procedure can be applied for $p_{MP} > p_{IP}$ r_{MP} : Pearson multiple correlation coefficient;		df: degrees of freedom;	ing entiter 14sher of Student distribution),			
correlation	$\Gamma_{MP} = \Gamma(1, 1), p_{MP} = p(\Gamma_{MP}, \Pi, \alpha_1 = \Pi)$	p_{SP} : probability of no linear dependence betw		$i \neq j$ (i < j is enough);				
analysis		a larger p_{MP} (usually > 5%) leads to rejecting		- / J (J				
Qualitative vs.	$r_{MS} = \rho(Y, \hat{Y}), p_{MS} = p(r_{MS}, m, df = n)$	r_{MX} : multiple qualitative correlation coefficie		ρ: Spearman ranks correlation coef	ficient:			
quantitative	$r_{M\tau a} = \tau_a(Y, \hat{Y}), p_{M\tau a} = p_Z(r_{M\tau a}, m, df = n)$	p_{MX} : probability of no linear dependence betw		τ_a : Kendall tau-a ranks correlation				
analysis	$r_{M\tau b} = \tau_b(Y, \hat{Y}), p_{M\tau b} = p_Z(r_{M\tau b}, m, df = n)$	a larger p_{MX} (usually more than 5%) leads to rejecting of MLR equation r_{MSP} : multiple semi-quantitative correlation coefficient; p_{MSP} : probability of no linear semi-quantitative dependence between Y and \hat{Y} ;		τ_b : Kendall tau-b ranks correlation coefficient;				
-	$r_{M\tau c} = \tau_c(Y, \hat{Y}), p_{M\tau c} = p_Z(r_{M\tau c}, m, df = n)$			τ_c : Kendall tau-c ranks correlation	τ_c : Kendall tau-c ranks correlation coefficient;			
	$r_{M\Gamma}=\Gamma(Y,\hat{Y}), p_{M\Gamma}=p_Z(r_{M\Gamma},m,df=n)$			Γ: Goodman-Kruskal ranks correla				
	$r_{MSP} = \sqrt{r_{MS} \cdot r_{MP}}$	a larger p_{MSP} (usually > 5%) lead to rejecting		Pz: probability of wrong model (us				
Leave-one-out	$r_{cv-loo}=r(Y, \hat{\hat{Y}}), p_{cv-loo}=p(r_{cv-loo},m,df=n)$	r_{cv-loo} : leave-one-out cross-validation correlat		$\hat{\hat{\mathbf{Y}}} = (\hat{\hat{\mathbf{Y}}}_k, k = 1n); \hat{\hat{\mathbf{Y}}}_k$ results from	the following algorithm:			
cross-validation	-cv-100 -(1, 1), pcv-100 p(-cv-100,, cu)	p _{cv-loo} : probability of no predictive linear model;		: Pamova malagula "k" from sampla:				
analysis		a larger p_{MP} (usually > 5%) leads to rejecting	ng of MLR equation as predictive line	$\dot{\mathbf{T}}$ + Then W:=Y \Y_k; MDFW_i = MDF_i				
		model;		÷ Apply MLR: $\hat{W} = \sum_i b_i MDFW_i$; b_i : real coefficients (MLR coefficients); \hat{W} estimator of W				
				$\div \hat{W} \text{ predictor for } Y_k: \hat{Y}_k = \Sigma_i b_i \text{MDF}_i(k)$				
Training vs. test	$r_{\text{training}} = r(Y _{\text{training}}, \hat{Y} _{\text{training}}),$	r: correlation between measured (V) and estimated $(\hat{\mathbf{Y}} \dots)$ into training					
experiment	$p_{\text{training}} = p(r_{\text{training}}, m_{\text{training}}, df = n)$	$\label{eq:rtraining} \begin{array}{l} r_{training}: \mbox{ correlation between measured } (Y _{training}) \mbox{ and estimated } (\hat{Y} _{training}) \mbox{ into training subset;} \\ p_{training}: \mbox{ probability of no linear dependence into training subset;} \\ r_{test}: \mbox{ correlation between measured } (Y _{test}) \mbox{ and predicted } (\hat{Y} _{test}); \\ p_{test}: \mbox{ probability of the no predictive ability of the MLR equation;} \\ a \mbox{ larger } p_{test} \mbox{ (usually > 5\%) combined with a small enough } p_{training} \mbox{ (usually < 5\%) } \\ \mbox{ leads to rejecting of MLR equation as predictive linear model;} \end{array}$			e sample after removing of the test set (usually of size of			
experiment	$r_{\text{test}} = r(Y _{\text{test}}, \hat{Y} _{\text{test}}),$			2m/3); m _{test} - size of test subset of the sample; m _{training} - size of training subset of the sample;				
	$p_{test} = p(r_{test}, m_{test}, df = n)$							
				$m_{\text{training}} = m - m_{\text{test}};$ Y _{ltest} - measured activity/property for test subset;				
				$ \mathbf{Y} _{\text{training}}$ - measured activity/property for training subset; $\mathbf{Y} _{\text{training}} = \mathbf{Y} \setminus \mathbf{Y}_{\text{test}}$;				
				$\hat{Y} _{test}$ results from the following alg	orithm:			
					$\Sigma_i c_i MDF_i$; c _i : real coefficients (MLR coefficients obtained			
1				from training set); $\hat{\mathbf{Y}}$				
				$ \begin{array}{c} \div \hat{U} \text{ estimator for } Y _{\text{training}} \cdot \hat{Y} _{\text{training}}(k) \\ \div \hat{U} \text{ predictor for } Y _{\text{test}} \cdot \hat{Y} _{\text{test}}(l) = \Sigma_i \ell \\ \end{array} $	$K = \Sigma_i c_i MDF_i(K), K \in \text{training};$			
Correlated	$Z_{\text{Steiger}}(Y, \hat{Y}1, \hat{Y}2, df12)$	$Z_{\text{Steiger}} < Z(5\%) = 1.96$: hypothesis of correlated	\mathbf{I} correlations between the estimators $\hat{\mathbf{N}}$	$+$ 0 predictor for 1_{lest} . $1_{\text{lest}}(1) - 2_{i}$	$r_i(\mathbf{n})$ $\mathbf{n} \in \mathbf{test},$ $\mathbf{n} = \mathbf{n}(\hat{\mathbf{V}}_1)$):			
correlations	$\Sigma_{\text{Steiger}}(1, 11, 12, 0112)$	and \hat{Y}_2 cannot be rejected with a confidence		df2: model 2 degrees of freedom (r				
analysis		Z_{Steiger} can serve for comparing of two MDF-		d12: model 2 degrees of freedom (m-n($Y 2$)); df12=min(df1,df2)-3; Z _{Steiger} computes from r(Y,Ŷ1), r(Y,Ŷ2), r(Ŷ1,Ŷ2), and df12;				
		Z_{Steiger} can serve for comparing of a MDF-SP						
EXPERIMENTAL: F	ollowing online applications were deve		T T T T T T T T T T T T T T T T T T T	Suger 1 1 1 1 1 1 1 1 1				
		_browse_or_query.php?database=MDFSARs/	(1) Simple correlation analysis; Inter	-correlation analysis; Multiple correl	ation Compute the Z value associated with the existence of correlated correlations			
1			analysis	, , , , , , , , , , , , , , , , , , ,	using the model proposed by Steiger [Steiger, J.H. Tests for comparing elements of a correlation matrix. Psychological Bulletin 1980, 87, 245-251.].			
ttp://l.academicdir	ect.org/Statistics/linear_dependence/		(2) Qualitative vs. quantitative analysi					
ttp://l.academicdirect.org/Chemistry/SARs/MDF_SARs/loo/		00/	(3) Leave-one-out cross-validation and	lysis	r1-2			
ttp://l.academicdirect.org/Chemistry/SARs/MDF_SARs/qsar_qspr_s/		(4) Training vs. test experiment	*	r2-3: N				
ttp://l.academicdir	ect.org/Statistics/tests/Steiger/	••	(5) Correlated correlations analysis		Submit Query			
napshots of the ap	plications (1)-(4) are presented in the ta	ble below:						
	(1)	(2)	(3)		(4)			
Up Browse o	r Query MDF SARs by sets. 🐣 🛛	p Leave one out analysis require a tabutate	ed <u>Up</u>	- <u>Up</u>				
	data	in html format as input data with followin		nt correlations between given				
Browse		column labels:	data columns. Computes correlation	n coefficients (Pearson, The exp	select a data file from the list of available data. periment will performe a random split of experimental data i			
IChr10_		row labels:	Spearman, Kendall, Gamma), cum	nulative distribution ratios (F, two set	ts: "trainig set" and "test set".			
[ICIIII0]		independent variables - first set of column	t, Z) and associated probabilities of		SAR/QSPR model are calculate using the data from training			
		estimated dependent variable - following		The ob	tained QSAR equation are apply then on both sets, in order to			
		column;	id d IP d IR d Cr d RSD d Volum 57 1.27 0.70 1.75 3.33 160.00	calcula	ate statistical parameters.			
Query		dependent variable;	79 0.97 0.61 5.49 2.57 160.00	19654.50	t Submit Query			
		predicted variable - last column;	80 0.97 0.60 2.81 2.47 137.00 81 0.88 0.58 2.61 2.39 146.00					
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		Browse Submit Query	Submit Query					
	1							
ESULTS:	Man	TERIALS:	MDF-SPAR completion: MDF Calc	ulator & MDF Predictor				
		hydrophobicity on Hessa et al. scale [¹⁰] of						
		en standard amino acids was the property of	Distance operator 7 Atom	c property: 6 Int	teraction model:			
spectively proved	to has estimated and predictive fifted	en standard amino acids was the property of						

$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	activity of the new compounds, even if this (these) compound(s) were not yet synthesized, in order to see if the new structure (virtual compound at this time) has or not improvements in desired property/activity. CONCLUSION	Descriptor (of interaction) formula: Distance, 'D' = d Inverted distance, 'd' = 1/d First atom's property, 'O' = p1 Inverted O, 'o' = 1/p1 Product of atomic properties, 'P' = p1p2 Inverted P, 'p' = 1/p1p2 Squared P, 'O' = p1p2^1/2 Inverted Q, 'q' = 1/p1p2^1/2 First atom's Property multiplied by distance, 'J' = p1d Inverted J, 'j' = 1/p1d Product of atomic properties and distance, 'K' = p1p2d Inverted K, 'k' = 1/p1p2d Product of distance and squared atomic properties, 'L' = d(p1p2 Inverted L, 't' = 1/p1p2d First atom's property potential, 'V' = p1/d First atom's property work, 'W' = p1/2/d Properties work, 'w' = p1p2/d Properties force, 'f' = p1p2/d^2 First atom's property force, 'F' = p1^2/d^3 Properties weak nuclear force, 'S' = p1^2/d^3 First atom's property strong nuclear force, 't' = p1^2/d^4 Properties strong nuclear force, 't' = p1p2/d^4	1 Cor Cor Cor Cor Cor Cor Cor Cor Cor Cor	ation criteria: ments, m gments, M	ns), B Int(atoms), F	2 n operator: ge). 1
difference	proved to be a very good tool for design of chemical compounds.			ance based fragments, D sed fragments, P	Logarithm of I, I	
	·	(6) MDF Calculator			IDF Predictor	
 ¹ LP Hammett, The Effect of Structure upon the Reactions ² C Hansch, A Leo, RW Taft, A Survey of Hammett Substi ³ Bolboacă SD, Jäntschi L, Modelling the Property of Com 	of Organic Compounds. Benzene Derivatives, J Am Chem Soc tuent Constants and Resonance and Field Parameters, Chem Re pounds from Structure: Statistical Methods for Models Validati theoretical kinetics and mechanics (in German), Zeitschrift für	ev 1991;91(2):165-195. on, Env Chem Lett DOI 10.1007/s10311-007-0119-9.	ect (2006-2008)). • <u>Up</u> Predict activity • a learning set and • a set of previous • any molecule sub Learning set:	l obtained MDF S	

The model with one and two descriptors,	fifteen standard amine soid area the mean at a f	Distance operator 7	Atomic property:	6	Interaction model:		1		
respectively proved to has estimated and predictive	fifteen standard amino acids was the property of	Topological distance, t	Cardinality, C		Rare model and resultant relativ	ve to fragment's hr	ad B		
abilities: $\hat{Y} = 0.59 \pm 3MDB = 0.5.852 = E_{-}(1)$	interest.	Geometrical distance, g	Count of directly bounded hidroge	n's, H	Rare model and resultant relativ				
$\hat{Y}_{mono} = -0.58 + iMDRoQg \cdot 8.53 Eq(1)$	The experimental values of hydrophobicity were	Relative atomic mass, M			Medium model and resultant relative to fragment's head, M				
$\hat{Y}_{bi} = -1.36 + iMDRoQg \cdot 6.03 + ISPDwQg \cdot 0.08 Eq(2)$	as follows: alanine (0.11), asparagine (2.05),		Atomic electronegativity, E		Medium model and resultant rel				
The application of the parameters presented in the	aspartate (3.49), cysteine (-0.13), glutamine		Group electronegativity, G		Dense model and resultant rela				
table bellow leads to the results presented bellow:	(2.36), glutamate (2.68), glycine (0.74),		Partial charge, Q		Dense model and resultant rela	tive to convention	al origin, d		
	isoleucine (-0.6), leucine (-0.55), lysine (2.71),	Descriptor (of interac	tion) formula:		Molecular overall supe	mosing form	ula		
Param. Eq(1) Eq(2)	methionine (-0.1), phenylalanine (-0.32), serine		lion) formula.			rposing rorm	iuia.		
$r_{SP}; p_{SP}$ 0.9514; 5.1·10 ⁸ \checkmark 0.8806; 1.5·10 ⁻⁵ \checkmark	(0.84), threonine (0.52), and valine (-0.31).	Distance, `D` = d			Cond., smallest, m		2		
$r_{MP}; p_{MP}$ n.a. 0.9238; 6.2·10 ⁹ \checkmark		Inverted distance, `d` = 1/d			Cond., highest M		Ľ		
$r_{IP}; p_{IP}$ n.a. 0.7726; 7.3·10 ⁴ \checkmark	Drug Design 🕨	First atom's property, `0` = p1			Cond., smallest absolute, n Cond., highest absolute, N				
$r_{MS}; p_{MS} = 0.9429; 1.4 \cdot 10^{-7} \checkmark 0.9643; 7.1 \cdot 10^{-9} \checkmark$	This facility of MDF-SAR allows that having:	Inverted 0, `o` = 1/p1 Product of storaic proportion	'P' - p1p2		Avg., sum, S				
$r_{MTa}; p_{MTa} = 0.8286; 1.7 \cdot 10^{-5} \checkmark 0.8857; 4.2 \cdot 10^{-6} \checkmark$	\dot{A} set of compounds of interest with known	Product of atomic properties, `P` = p1p2 Inverted P, `p` = 1/p1p2		Avg., sun, 3 Avg., average, A					
$r_{Mrtb}; p_{Mrtb} = 0.8286; 1.7 \cdot 10^{-5} \checkmark 0.8857; 4.2 \cdot 10^{-6} \checkmark$	values of property/activity and MDF-SARs	Squared P, 'Q' = p1p2^1/2			Avg., S/count(fragments), a				
	obtained, validated, and stored into the	Inverted Q, 'q' = 1/p1p2^1/2			Avg., Avg.(Avg./atom)/count(ato	ims), B			
	database;	First atom's Property multiplie	ed by distance, `J` = p1d		Avg., S/count(bonds), b	a			
	÷One of more similar/alike with selected	Inverted J, 'j' = 1/p1d	and a stand database to the sound of the second standing of the seco		Geom. product P				
$r_{MSP}; p_{MSP} = 0.9471; 8.7 \cdot 10^8 \checkmark 0.9714; 1.7 \cdot 10^9 \checkmark$		Product of atomic properties and distance, `K` = p1p2d Inverted K. `k` = 1/p1p2d			Geom., mean, G Geom., P^1/count(fragments), g				
$r_{cv-loo}; p_{cv-loo} = 0.8744; 9.6 \cdot 10^{-7} \checkmark 0.9158; 1.7 \cdot 10^{-7} \checkmark$	compound(s) set by made of:								
$r_{tr}^{2}; p_{tr} = 0.8619; 1.0 \cdot 10^{-4} \checkmark 0.9572; 1.6 \cdot 10^{-5} \checkmark$	$r_{\rm tr}^2; p_{\rm tr} = 0.8619; 1.0 \cdot 10^4 \checkmark 0.9572; 1.6 \cdot 10^5 \checkmark \circ \text{MDF-SAR equation (MDF predictor)};$						Geom., Geom.(Geom./atom)/count(atoms), F		
$r_{ts}^{2}; p_{ts} = 0.9862; 4.3 \cdot 10^{-3} \checkmark 0.9629; 4.8 \cdot 10^{-2} \checkmark$	 building (with HyperChem) of topological 					Geom., P ¹ /count(bonds), f			
Z _{Steiger} ; p 1.7847; 0.074 ×	(2D) and geometrical (3D) through same			Harm., sum, s Harm., mean, H					
	choices as were build the selected set	First atom's property work, 'V			Harm., s/count(fragments), h				
m_{training} Eq(1) = 10 (valine, cysteine, aspartate,	to obtain predicted value(s) for the property /	Properties work, `w` = p1p2/c			Harm., Harm.(Harm./atom)/coun	t(atoms) I			
nethionine, isoleucine, threonine, glutamate, activity of the new compounds, even if this				Harm, s/count(bonds), i					
asparagine, glutamine, alanine); m _{test} = 5 amino	(these) compound(s) were not yet synthesized, in	Properties force, 'f' = p1p2/d'		100		T in anti-			
acids.	order to see if the new structure (virtual	First atom's property weak nu			3		on operator:		
m_{training} - Eq(2) = 10 (cysteine, alanine, threonine,	compound at this time) has or not improvements	Properties weak nuclear forc		-		Identity (no cha	inge), l 1		
leucine, glycine, glutamate, serine, aspartate,	in desired property/activity.	First atom's property strong r		Fragm	entation criteria:	Inversed I, i			
valine, phenylalanine)	Conclusion	Properties strong nuclear for	ce, `t` = p1p2/d^4		fragments, m	Absolute I, A			
where: \square = statistically significant & \blacksquare = no					fragments, M	Inversed A, a Logarithm of A,	î.		
difference	proved to be a very good tool for design of			Szeged	distance based fragments, D	Logarithm of I, I			
unreferice	chemical compounds.			Cluj path	based fragments, P	[Loganani ori, i			
	chemical compounds.	(6) MDF Calculator		(7) N	MDF Predictor			
ACKNOWLEDGEMENTS: [MDF] The MDF project wa	us supported through ET36 research project (2005-2007). [M	(DF-SAR) The MDF-SAR of M	DF is support through ET108 proje	ct (2006-2	008)	1 1			
	s supported unough 2100 research project (2000 2007). [h.		in in support anough hiros proje	2000 2	- <u>op</u> Fredict activity				
					• a learning set and				
	of Organic Compounds. Benzene Derivatives, J Am Chem Soc 1				 a set of previous 				
	uent Constants and Resonance and Field Parameters, Chem Rev				 any molecule su 	bmitted as HIN f	file by the user.		
	ounds from Structure: Statistical Methods for Models Validation		10311-007-0119-9.		Learning set:				
	heoretical kinetics and mechanics (in German), Zeitschrift für P	hysik, 1927;43(3-4):172-198.			Dearning set.				
⁵ Jäntschi L, MDF - A New QSAR/QSPR Molecular Descrip					15aacids 👻 Submit C	Query			
	tition Coefficient of Substituted Phenols by the Use of Structure								
	tivity Relationships 1. Review of the Methodology, Leonardo El								
	lar Descriptors Family on Structure Property/Activity Relationsh tetention Times of Polychlorinated Biphenyls: from Structural In			1157					
	rsson H, Nilsson I, White SH, von Heijne G, Recognition of trar				381				
nessa 1, Kini fi, Dininaler K, Lununi C, Boekel J, Ander	sson n, misson i, white sn, von neijne G, kecognition of trar	ismemorate tiences by the endop	asine redculum translocon, Nature 20	05;455:577	-301.				